

induced learning disability.”

Similarly, even though there is evidence that sleep is important for learning and memory, teenagers are notoriously sleep-deprived. Studying right before bedtime can help cement the information under review, Jensen notes. So can aerobic exercise, says Urion, bemoaning the current lack of physical-education opportunities for many American youths.

Teens are also bombarded by information in this electronic age, and multitasking is as routine as chatting with friends on line. But Jensen highlights a recent

study showing how sensory overload can hinder undergraduates’ ability to recall words. “It’s truly a brave new world. Our brains, evolutionarily, have never been subjected to the amount of cognitive input that’s coming at us,” she says. “You can’t close down the world. All you can do is educate kids to help them manage this.” For his part, Urion believes programs aimed at preventing risky adolescent behaviors would be more effective if they offered practical strategies for making in-the-moment decisions, rather than merely lecturing teens about the behav-

iors themselves. (“I have yet to meet a pregnant teenager who didn’t know biologically how this transpired,” he says.)

By raising awareness of this paradoxical period in brain development, the neurologists hope to help young people cope with their challenges, as well as recognize their considerable strengths.

~DEBRA BRADLEY RUDER

FRANCES JENSEN E-MAIL ADDRESS:

[frances.jensen@childrens.harvard.edu](mailto:frances.jensen@childrens.harvard.edu)

DAVID URION E-MAIL ADDRESS:

[david.urion@childrens.harvard.edu](mailto:david.urion@childrens.harvard.edu)

MYSTERIES OF MATH

# Proof Positive

**A**S ACADEMICS work to understand the architecture of the universe, they sometimes uncover connections in mysterious places. So it is with Smith professor of mathematics Richard L. Taylor, whose work connects two discrete domains

of mathematics: curved spaces, from geometry, and modular arithmetic, which has to do with counting. Taylor has spent his career studying this nexus, and recently proved it is possible to use one domain to solve complex problems in the other. “It just astounded me,” he says, “that there should be a connection between these two things, when nobody could see any real reason why there should be.”

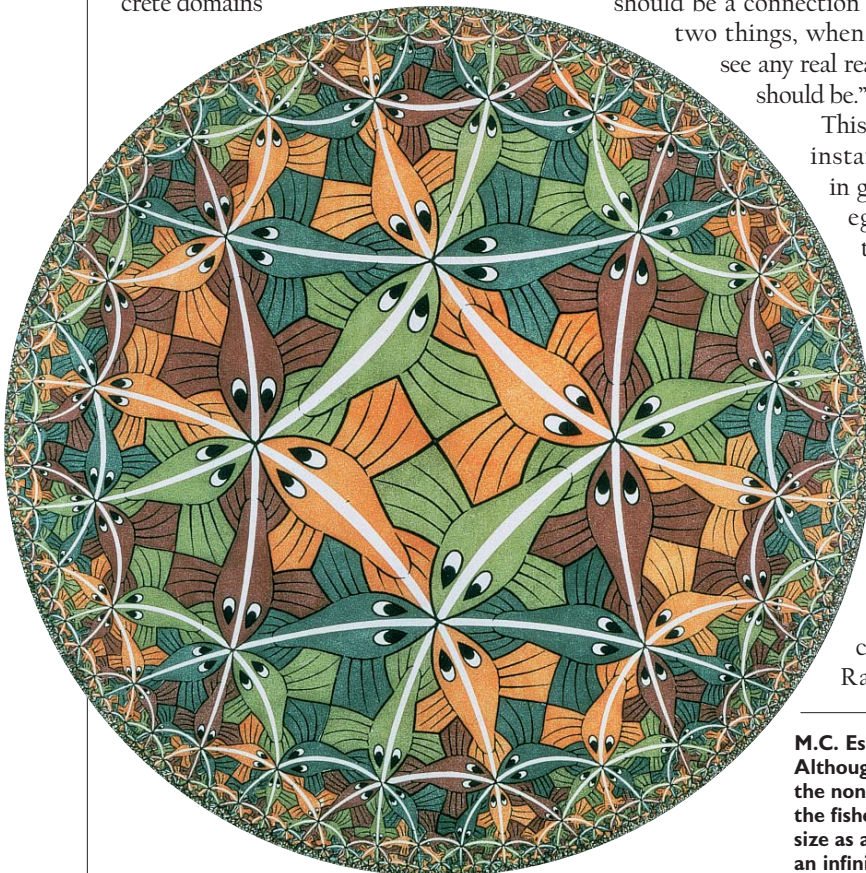
This is not the first instance of finding in geometry an elegant explanation for a seemingly unrelated phenomenon. Scholars during the Renaissance, seeking a mathematical basis for our conceptions of beauty, fingered the so-called Golden Ratio (approx-

mately 1.6 to 1). Some analyses find the ratio in structures—most famously the Parthenon—built centuries before its first written formulation. More recently, scientists have found that the faces people find most beautiful are those in which the proportions conform most closely to the ratio.

The geometry-arithmetic connection explored by Taylor solves another puzzle that has enticed mathematicians across centuries. In 1637, French mathematician Pierre de Fermat scrawled in a book’s margin a theorem involving equations like the one in the Pythagorean theorem ( $a^2 + b^2 = c^2$ ), but with powers higher than two. Fermat’s theorem said such equations have no solutions that are whole numbers, either positive or negative. Go ahead, try—it is impossible to find three integers, other than zero, that work in the equation  $a^3 + b^3 = c^3$ .

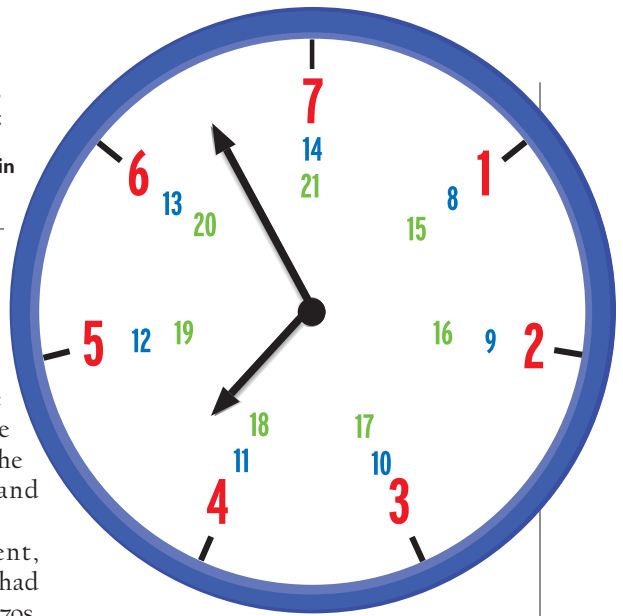
The French mathematician also wrote that he had discovered a way to prove this—but he never wrote the proof down, or if he did, it was lost. For more than 350 years, mathematicians tried in vain to prove what became known as Fermat’s Last Theorem. They could find lots of examples that fit the pattern, and no counterexamples, but could not erase all doubt until Princeton University mathematician Andrew Wiles presented a proof in 1993.

His discovery made the front page of the *New York Times*, but six months later,



**M.C. Escher’s Circle Limit III illustrates the concept of hyperbolic space. Although the fish appear to get smaller toward the edge of the image, in the non-Euclidean world of hyperbolic geometry, the white lines along all the fishes’ spines are actually the exact same length. Each fish is the same size as all the others, and an inhabitant of this world would have to walk an infinite distance to reach the circle’s edge.**

This type of unconventional clock is commonly used to illustrate problems in modular arithmetic. This particular clock has 7 “hours”; each “hour” position holds a set of numbers that are congruent modulo 7, or separated by multiples of 7. For one particularly elusive set of equations, earlier mathematicians located an ingenious but labor-intensive solution method in modular arithmetic. Richard Taylor’s work finds a shortcut in geometry.



the *Times* reported that another mathematician had found a mistake in the new proof. In the spotlight and under pressure, Wiles called on Taylor, his former doctoral student, for help. Together, they published the corrected proof in 1995, and Wiles won the Fermat Prize for mathematics that year. Taylor won the same prize in 2001, and last year shared the prestigious, million-dollar Shaw Prize for

“It just astounded me that there should be a connection between these two things.”

proving the 1963 Sato-Tate Conjecture, which said that one could draw a curve that would predict the number of solutions for any equation describing an elliptic curve.

The conjecture itself was a brilliant insight, but applying it is labor-intensive—it relies on modular arithmetic, a domain of mathematics that studies the relationships between numbers, often illustrated using clock-like figures with unusual numbers of hours (i.e., not 12). Counting the number of solutions for an

equation required counting one’s way around the “clock” (see figure at right). In proving the conjecture, Taylor essentially mapped the directions for a massive shortcut. But the proof was, as Taylor puts it, “a nice by-product” of his main work on the connection between arithmetic and geometry.

Taylor’s Shaw Prize co-recipient, mathematician Robert Langlands, had been the first to realize, back in the 1970s, that instead of counting the number of solutions for these equations, it would be possible to get the same answer using geometry. Specifically, the shortcut lay in the symmetry of hyperbolic space, one of the mathematical concepts illustrated by the artist M.C. Escher. In this weird, warped world, pairs of parallel lines break all the laws of Euclidean geometry and bend away from one another, and lines that, to the human eye, look wildly different are actually the same length. The analogy commonly made is to a horse’s saddle: viewed from above, a saddle appears as an oval, but its circumference is unexpectedly long because it curves downward along the sides of the horse’s body and upward toward the horse’s head and tail.

Although Taylor teaches everything

from undergraduate calculus to advanced graduate courses, his primary research is work on a kind of foreign-language dictionary that translates between these two mathematical domains—between the language of clocks and that of saddles. Taylor describes the dictionary this way: “It says, ‘Here’s an equation. I’m going to give you a problem about symmetry that has the same behavior.’” He and other number theorists are busily filling in more entries to illustrate patterns and perhaps solve still-unidentified mathematical mysteries. “We’ve done A and B so far,” he says. “All the way up to Z is left.”

~ELIZABETH GUDRAIS

RICHARD TAYLOR E-MAIL ADDRESS:  
[rtaylor@math.harvard.edu](mailto:rtaylor@math.harvard.edu)

ANNALS OF DE-MINING

## Man, Mongoose, and Machine

**S**TANDING OUTSIDE a Sri Lankan army base in the spring of 2007, Thrishantha Nanayakkara mapped an entire minefield without once setting foot in it. Nanayakkara held a remote control and periodically made a note on his computer. A mongoose hitched to a robot did most of the work.

This unorthodox de-mining team avoids most of the traditional pitfalls. Metal detectors give too many false alarms, fooled by bullets or other debris. Hand-held ground-penetrating radar

machines are too expensive. Dogs weigh enough to trigger mines, injuring or killing themselves and their handlers.

Nanayakkara, a visiting scholar at the School of Engineering and Applied Sciences and a 2008-09 Radcliffe Institute fellow, picked an indigenous mongoose for its temperament, size (roughly 2.5 kilograms, light enough to step on a mine without detonating it), and sense of smell (able to detect explosives three meters away). He equipped his robot (roughly a meter long and half a meter wide) with a

harness to keep the mongoose under control and a video camera to record its findings. Although the mongoose walks a few feet ahead, the robot with its eight metal legs sets the pace. During the test run, the pair went back and forth across a 10-by-10-meter plot, stopping whenever the mongoose detected a mine, which it indicated by sitting up (as it was trained to do). In a morning’s work, the mongoose found every mine.

The land mines in Sri Lanka—and other war-torn nations—are both physi-