Mathematics from the Inside Out

A practitioner on the human enterprise of pure mathematics

by AVNER ASH

In 1940, in the shadow of World War II, G.H. Hardy, one of the great mathematicians of the twentieth century, published a short book called A Mathematician’s Apology. Hardy argued that the great bulk of higher mathematics—and in particular his branch of it, number theory—while useless, derives its worth from its enduring truth and beauty.

Hardy was dogmatic in his thinking and style. For example, here is what he said about his own endeavor in writing his book:

If then I find myself writing, not mathematics but “about” mathematics, it is a confession of weakness, for which I may rightly be scorned or pitied by younger and more vigorous mathematicians. I write about mathematics because, like any other mathematician who has passed sixty, I have no longer the freshness of mind, the energy, or the patience to carry on effectively with my proper job.

The world is now much changed. Advanced number theory is crucially applied to cryptography, enabling, among other things, fairly secure transmission of credit card, financial, and other data over the Internet—and presumably the spying activities of the National Security Agency. The attachment to Truth and Beauty felt by Hardy has been shaken to its foundations by postmodern thought.

It is high time for Michael Harris’s book, mathematics without apologies. Note, for starters, the lack of capital letters in the title. Harris, also a great mathematician past the age of 60 (he is a professor at Columbia and the Université Paris Diderot), has written a very interesting, very peculiar, and very timely essay on the “what” and “why” of pure mathematics.

Unless you are also a mathematician, you are not likely to have much understanding of the inner professional life of a mathematician, even if you have studied the subject for many years in school. Why is that? And should you care? If you do care, then this book may help explain why the essence of mathematics is so foreign to most people, how the practice of mathematics feels to the working research mathematician, and what attracts mathematicians to their work.

By mixing memory and desire with mathematics, Harris has provided fresh responses to all the standard questions: “What do mathematicians do, actually?”

Yet Only the Animals is apolitical. It engenders empathy, shame, and sadness, but also wonder at these spirited creatures. They face what life and death bring with enviable presence of mind and body, as visceral beings. “What choice did she have,” asks the parrot in Beirut, “but to hook my cage to the awning overhead and leave as quietly as she could, before I realized I was alone?”

“I am very aware that we are all creatures who suffer together, and that existence is hard for us all,” Dovey reflects. “There is something, also, about the bond we have with animals, the care and connection that we don’t appreciate or see the magic in as much as we should.” Animal guides, she points out, have graced children’s literature throughout the world. “They are like oracles, there at our very earliest attempts to build empathy and imagination.” And that takes work, she says: those capacities “do not come automatically, in the sense that cruelty is a failure of the imagination. Something happens in reading through these animal guides that is very tied up in what it means to be a good human being.”

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“Hasn’t all mathematics already been discovered?” “Do you just add up numbers all day?” “Can you explain to me a little about your research?” Only a mathematician could have written these answers from the inside out. Harris also has a well-furnished vocabulary from outside mathematics with which to theorize about the sociology and psychology of mathematical practice.

An analogy between mathematics and religion is apt. In both, problems come first: in religion, problems of life; in mathematics, problems about patterns of numbers and shapes. Then a tradition, rooted in a human community, shapes responses to the problems, formulates refinements of the problems, and sometimes raises whole new issues.

Harris takes Max Weber’s concept of charisma as basic in understanding the role of the “great” mathematicians in history and in our times. These “giants” both shape the field as to contents and methods and inspire their lesser contemporaries and all who follow to healthy emulation. Harris thus understands mathematics as a result of human beings’ mathematical activity, observed from the viewpoint of an amateur social scientist. He treats any ultimate connection between mathematics and “truth” either skeptically or elides it. As for “beauty” or “utility,” he explicitly denies that the pursuit of either can be the fundamental motivation of a pure mathematician.

This approach embodies a postmodern perspective, which avoids ultimates. Instead of Hardy’s high-minded dictates, Harris focuses on mathematical research as a lived experience. His aim is to describe to a general audience the human experience in pursuit of pure mathematics.

As a result, he tends to undercut most mathematicians’ conviction that their subject matter is objective and in fact “True” with a capital T. Harris calls that concept “Mathematics” with a capital M, which leads philosophically or logically minded people to seek irrefragable “foundations” for the subject. He regards “Mathematics” as a figment of the imagination and instead places the stress on “mathematics” as the output of a tradition mediated by the series of actual human mathematicians, leaving the question of truth—capitalized or not—somewhat vague.

From this sociological starting point, Harris discusses a wide variety of topics, including the role of mathematics in capitalist finance (especially in the crash of 2008), and the relation between mathematics and the human body, with special attention to love and sex. (This treatment begins with Archimedes in his bath, exposes the rather sexy image of the mathematician in the Enlightenment, and ends with a discussion of some mystical Russian mathematicians of the early twentieth century.)

The core chapters of the book describe how intuition functions in advanced mathematical research. Outsiders often think that mathematics is all cut-and-dried logic. Not so. Harris shows how many of the most important ideas that drive the subject forward are nebulous, suggestive, not strictly logical at all.

There is a powerful difference between lived, addictive mathematics and its dead logical effigy. In logic, any correct argument is as good as any other. But in mathematical practice, different sorts of arguments have different psychological and cognitive appeal. For instance, “tricks” are surprising arguments that go “from peak to peak” and somehow avoid a deeper study that would reveal more of the terrain or structures that lie beneath the theorems discovered by “tricks.”

Harris rounds out this argument with an autobiographical account of a three-month period of research inspired by a fuzzy dream. The dream hinted at an unexpected way to study certain profound number-theoretic properties of groups of matrices by transferring well-known techniques from related areas of algebra and geometry to the case at hand. The account shows how some mathematical research is influenced by unconscious thought processes—perhaps, Harris speculates, driven by envy—even as he documents discussions with other people, e-mails, research papers, lectures, and seminars. Unfortunately, despite the personal approach, this chapter and its footnotes present the mathematical material at a very high level, with all the jargon and ideas intact, and my guess is, this may sound mostly like white noise for readers lacking background in the field. That’s regrettable, because it is an authentic account of research performed at a high level of competency.
and creativity, so one hopes a sense of the
ideas’ complexity and connections will
come across nevertheless.

Other, interspersed chapters of math-
ematical exposition explain, in an enter-
taining and elementary way, some prob-
lems in number theory extending from
ancient Greek examples to the most up-
to-date. These ideas, problems, and theo-
rems include basic properties of prime
numbers, solutions of algebraic equa-
tions in one and two variables with inte-
gral coefficients (rational, irrational, and
transcendental), and congruences among
the integral solutions. They culminate in
recently solved and currently unsolved
problems involving elliptic curves (cubic
equations in two variables.) These expla-
nations should be accessible to anyone
who knows high-school math.

IN TWO AND A HALF PAGES of the “after-
word,” Harris quotes or mentions Niet-
zsche, David Hilbert, Goethe, Wilde, Rich-
ard Strauss, Schiller, Kant, Schlegel, Novalis,
Felix Hausdorff, Mongré (Hausdorff’s liter-
ary pseudonym), L.E.J. Brouwer, Georg Can-
tor, Schopenhauer, Alexander Grothendieck,
Alexander Beilinson, Nikolai Luzin, Dmitri
Egorov, and Thomas Pynchon. For
better or worse, this gives an inkling of his
wide range—and of the breakneck speed at
which he operates. (Elsewhere in the book,
Harris often brings up the Vedas and later
Indian philosophy to give perspective on
the Western philosophical attitudes that
are his main subject. He also discusses Arab
mathematicians and briefly remarks on oth-
er non-Europeans, like Omar Khayyam and
Brahmagupta.)

I learned something new on almost ev-
ery page. For example, Harris endorses the
ancient Greek mathematician and philoso-
pher Eudoxus’s association of the pursuit
of mathematics with the pleasure it brings
practitioners. In his modern formulation,
Harris writes, “The short answer to the
‘why’ question is going to be that math-
ematicians engage in mathematics because
it gives us pleasure”—very much of a piece
with his larger account of mathematics
as human enterprise, rather than as any
greater quest for an abstract truth.

Comparing mathematical activity and
play, Harris says that “there is no need to
seek the meaning of mathematics else-
where than in the practice constituted by
tradition; and the telos of mathematics is to
develop this meaning as a way of expand-
ing the relaxed field [a field not subject to
normal pressures of material gain and pro-
ductivity...This book’s] goal is to convey...
what it is like to be a mathematician, freely
choosing a tradition to which to adapt,
not to serve the Powerful Beings of market
rationality nor the metaphysical Powerful
Beings of our own creation.”

Granting the book’s pleasures and in-
sights—there is rarely a dull moment—
Harris’s writing is at times choppy, jump-
ing from one level of discourse to another.
It can be hard to follow the nuances and
consequences and connections among the
ideas in their rapid flow.

With that caveat, I would recommend
mathematics without apologies to anyone curi-
ous about what it is like to be a modern
practitioner of this ancient field. You will
encounter a unique, unapologetic account
of the being (or Being?) of mathematicians.

Amer Ash ’71, Ph.D. ’75, is professor of math-
ematics at Boston College, specializing in number
theory. His forthcoming book Summing It Up
(with Robert Gross) will be published by Princeton
University Press.

A Broadcast Cornucopia
For 75 years, WHRB has moved beyond the “warhorses.”

by CRAIG A. LAMBERT

THERE may not be another radio
station in America that would air
a show like the one WHRB (95.3
FM) broadcast in February of 2013:
an hour and a half of music with no song lon-
ger than one minute. “It was the most stress-
ful 90 minutes of my life,” says Peter Menz
’15, a former rock director for the Record
Hospital department at Harvard’s WHRB,
who produced and deejayed the broadcast.
“A minute can seem like a very long time.
This was not one of those times. I had pulled
80 or 90 songs, and played 60 or 65”: a torrent
of music, with barely time in between to
announce titles and

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